# Proton structure, muon-proton interactions and the proton radius puzzle

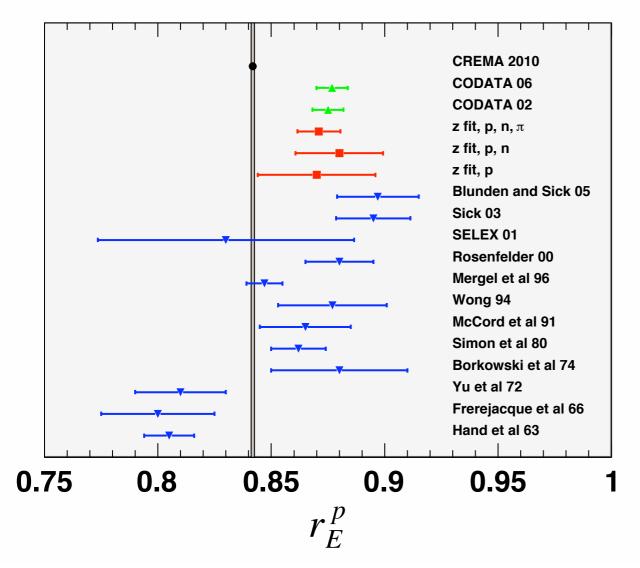
#### RICHARD HILL



Project X workshop 20 June 2012

Based on work w/ G. Paz, Phys.Rev.Lett. 107 (2011) 160402, Phys.Rev. D82 (2010) 113005

## the proton radius puzzle



- inferred from muonic H
- inferred from electronic H
- extraction from e p, e n scattering,  $\pi\pi NN$  data (this talk)
- previous extractions from e p scattering (as tabulated in PDG)

### This talk:

- formalism for proton/nuclear structure effects in hydrogenic bound states
- analyticity and nucleon form factors
- status of the proton radius puzzle(s)

comparison of muon anomalies:

	(g-2) <sub>µ</sub>	r <sub>E</sub> <sup>p</sup>
significance	3.6σ e+e- 2.4σ τ	5σ H spectroscopy I σ - 5σ ep scattering
hadronic uncertainties	hadronic vac. pol, light-by-light	charge radius, two-photon exchange
new physics/SUSY interpretation	≈√?	?

A basic problem in the measurement of fundamental parameters (or new physics?) A thorny (fun) problem at the interfaces of atomic, nuclear and particle physics.

"Data from muonic hydrogen are so inconsistent with the other data that they have not been included in the determination of  $r_p$  and thus do not have an influence on  $R_\infty$ " - CODATA 2010

"Until the difference between the  $e\,p$  and  $\mu\,p$  values is understood, it does not make much sense to average all the values together. For the present, we stick with the less precise (and provisionally suspect) CODATA 2006 value. It is up to workers in this field to solve this puzzle." - PDG 2011

"Testing of this result is among the most timely and important measurements in physics" JLab PAC, 2011

#### significant interest from multiple perspectives



spelov. May 2012. Note: \* Temporary entry \*; To appear in PRI

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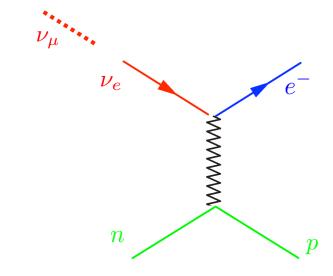
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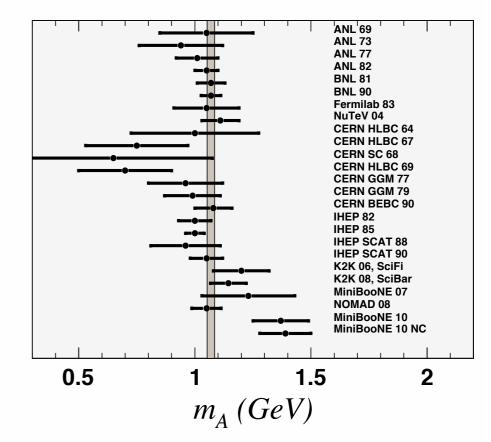
## aside: proton axial radius puzzle

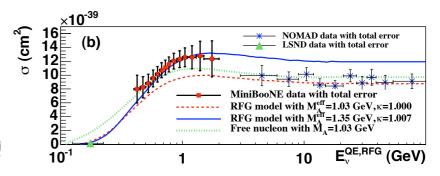
Charged current quasielastic scattering: basic signal process for neutrino oscillation



$$\langle p(p')|J_W^{+\mu}|n(p)\rangle \propto \bar{u}^{(p)}(p') \left\{ \gamma^{\mu} F_1(q^2) + \frac{i}{2m_N} \sigma^{\mu\nu} q_{\nu} F_2(q^2) \right\}$$

$$+ \gamma^{\mu} \gamma_5 F_A(q^2) + \frac{1}{m_N} q^{\mu} \gamma_5 F_P(q^2) \bigg\} u^{(n)} (q^2) \bigg\}$$





Considerable uncertainty and discrepancies in cross section from

- axial-vector form factor
- nuclear effects

$$F_A^{\text{dipole}}(q^2) = \frac{F_A(0)}{\left[1 - q^2/(m_A^{\text{dipole}})^2\right]^2} \cdot m_A \equiv \sqrt{\frac{2F_A(0)}{F_A'(0)}}$$

Both are essential to the interpretation of neutrino oscillation studies

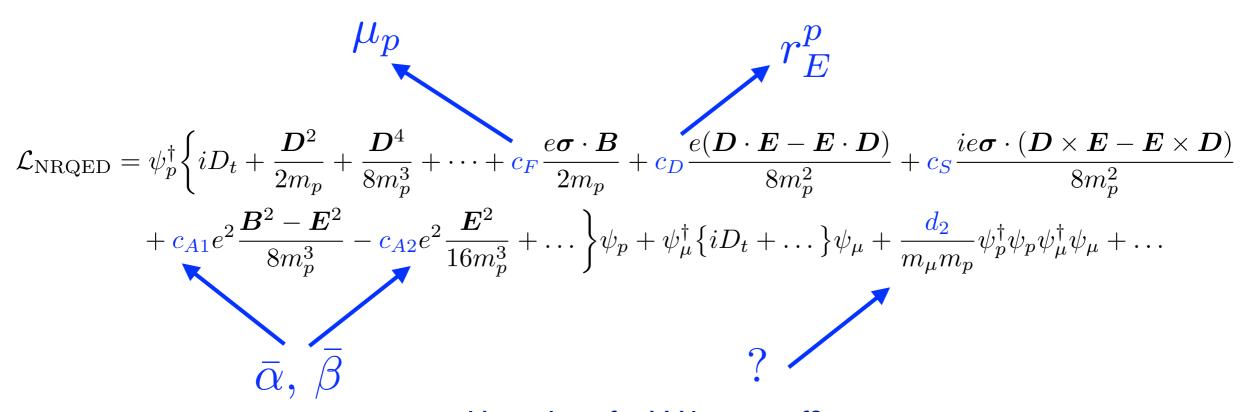
#### progress in theoretical tools:

- z expansion and dispersive analysis
  - free gift from field theory to analyze scattering data
  - wide range of applications: EM form factors, neutrino scattering, meson transitions, ...
- high orders of NRQED
  - define the charge radius and other proton structure corrections in presence of radiative corrections
  - model independent spectroscopic predictions in terms of scattering data

## NRQED and proton structure

energy, momenta << M: integrate out M

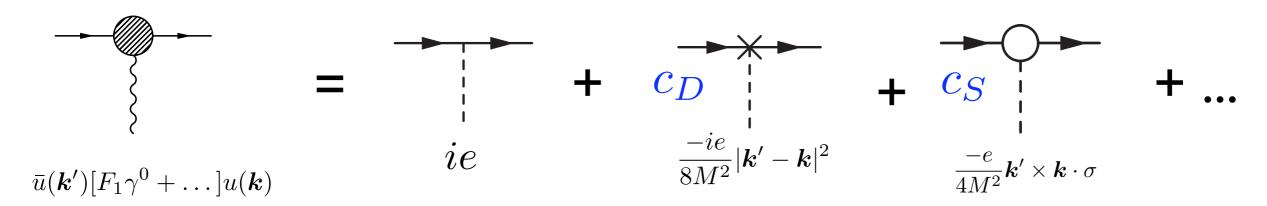
[Caswell, Lepage 1986]



proton structure appears as non-pointlike values for Wilson coefficients

- matching performed by computation, or comparison to experiment
- relativistic invariance (infinite dimensional realization of Poincare) implies constraints on coefficients:  $c_S = 2 c_F 1$ , etc.
- muonic hydrogen Lamb shift: need  $c_D$  through  $O(\alpha)$ ,  $d_2$  through  $O(\alpha^2)$

## NRQED part (1): vertex corrections



$$\langle k'|J_{\text{e.m.}}^{\mu}|k\rangle = \bar{u}(k')\left[\gamma^{\mu}F_{1}(q^{2}) + \frac{i}{2M}F_{2}(q^{2})\sigma^{\mu\nu}q_{\nu}\right]u(k)$$

#### Convenient to work in terms of Sachs basis:

$$G_E = F_1 + \frac{q^2}{4M^2} F_2$$
  $G_E(0) = 1$   
 $G_M = F_1 + F_2$   $G_M(0) = \mu_p \approx 2.793$ 

At tree level,

$$c_D = 1 + 8G'_E(0) \equiv 1 + \frac{4}{3}r_E^2$$

#### Need $c_D$ correct to $O(\alpha)$ :

$$G_E'(0) \equiv \frac{1}{6} (r_E^p)^2 + \frac{\alpha}{3\pi m_p^2} \log \frac{m_p}{\lambda}$$

(or equivalent IR finite observable)

#### Defines r<sub>E</sub> in presence of radiative corrections

- can extract from/compare to electronic hydrogen
- must use same definition in comparison to hydrogen data (more later)
- can extract from measured electron-proton scattering data



- Assume that two-photon exchange can be reliably subtracted (more later)
- Need to extrapolate to  $Q^2=0$  to find slope, and hence  $c_D$

## analyticity and form factor constraints

What functional form to use in extrapolating to  $Q^2=0$ ?

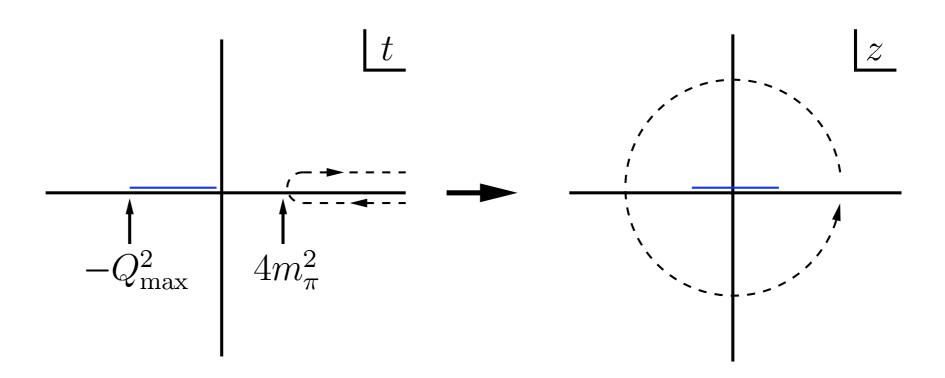
$$G_E=1+a_1q^2+a_2q^4+\dots$$
 [Simon et al 1980] radius of convergence < 4 m $_{\pi}^2$ 

$$G_E = \frac{1}{1 + a_1 \frac{q^2}{1 + a_2 \frac{q^2}{1 + \dots}}}$$
 [Sick 2003]

no control on parameters

<u>fundamental problem</u>: need larger Q<sup>2</sup> to increase statistics but then introduce sensitivity to more parameters (need even more statistics, ...)

analyticity:



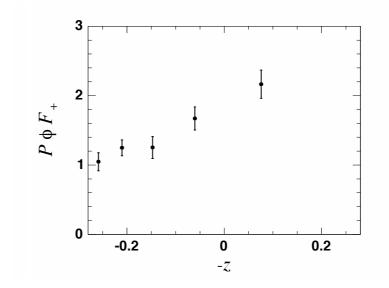
- extended to complex values of  $t=q^2$ , form factor is analytic outside cut in t plane

$$z(t, t_{\rm cut}, t_0) = \frac{\sqrt{t_{\rm cut} - t} - \sqrt{t_{\rm cut} - t_0}}{\sqrt{t_{\rm cut} - t} + \sqrt{t_{\rm cut} - t_0}}$$

$$4m_{\pi}^2 \text{ (isovector channel)} \qquad \text{point mapping to z=0} \text{ (scheme choice)}$$

- sums simple Taylor expansion, ensuring convergence through entire physical range

## techniques familiar from meson transition form factors, e.g. $B \rightarrow \pi$



[Bourrely et al 1981]

[Boyd, Grinstein, Lebed 1995]

[Lellouch et al 1996]

[Arnesen et al 2005]

[Becher, Hill 2006] ....

- basic idea: small expansion parameter (z), order unity expansion coefficients

$$G(q^2) = \sum_{n=0}^{\infty} a_n z(q^2)^n$$

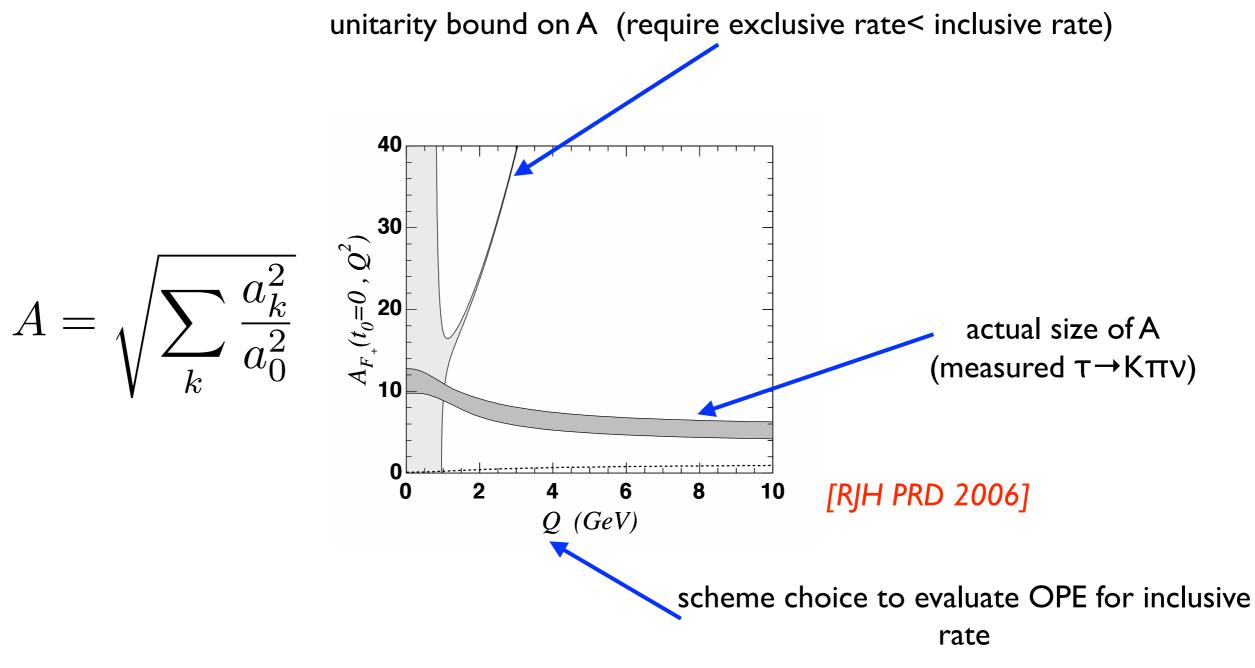
- in fact, a little better, since can show

$$\sum_{n=0}^{\infty} a_n^2 < \infty \quad \Longrightarrow \quad$$

## an smaller for large n

- a form factor not in this class is in violation of QCD; conversely, stronger constraints require more knowledge (that should be made explicit)

For the cognoscenti, the real power of the expansion is based on observation of O(1) coefficients, not unitarity bounds. Example from  $K \rightarrow \pi$  vector form factor

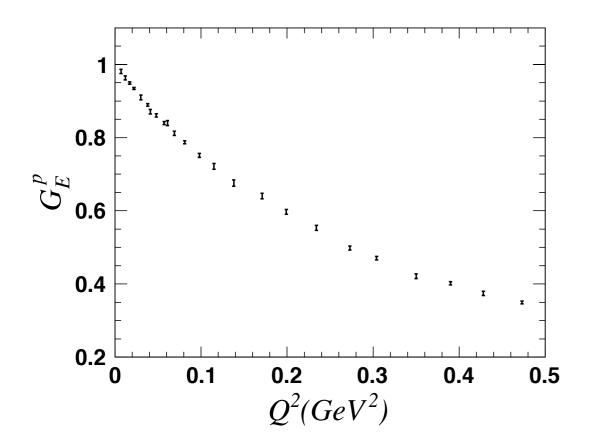


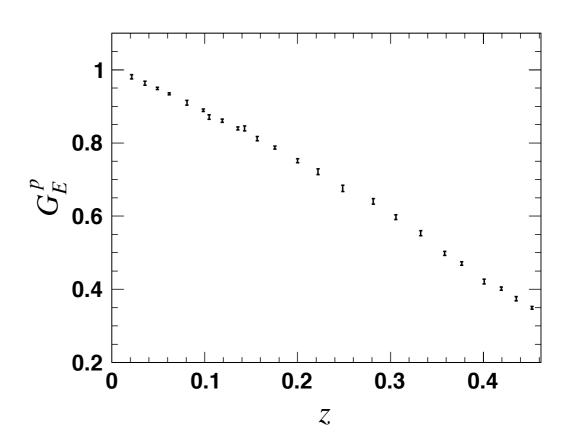
 $\Rightarrow$  Unitarity bound either uncertain (low Q) or overestimates bound (high Q)

University of Chicago

For nucleon form factors, unitarity even less relevant, as dominant dispersive contribution to form factors is from states below NN threshold

- study of vector dominance models,  $\pi\pi$  approximation to isovector form factors: expect O(I) is really order I (e.g. not I0)



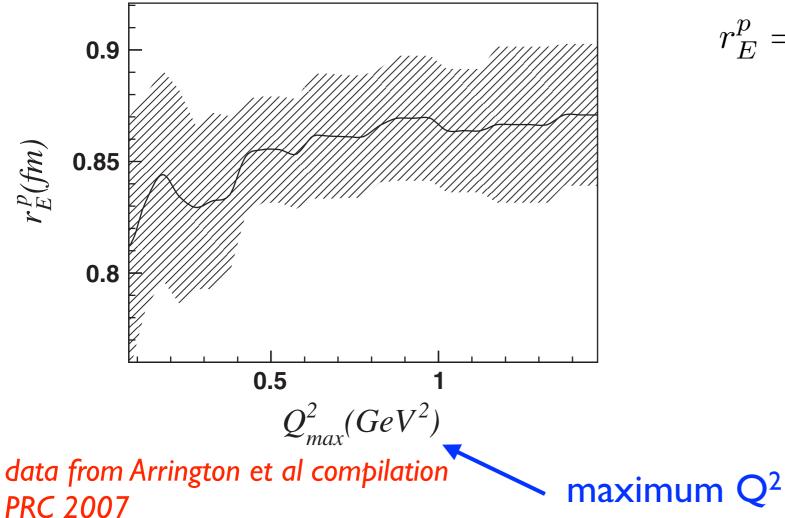


- more concretely, fits to data yield

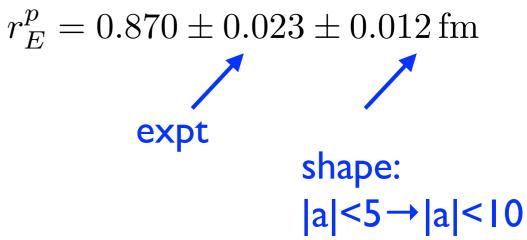
$$a_0 \equiv 1$$
,  $a_1 = -1.01(6)$ ,  $a_2 = -1.4^{+1.1}_{-0.7}$ ,  $a_3 = 2^{+2}_{-6}$ 

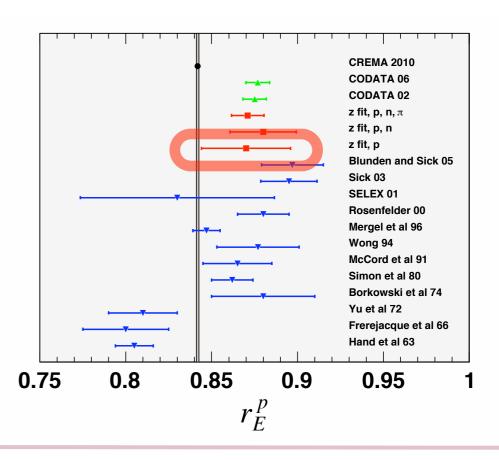
to assign error, constrain coefficients <5 (conservative) or</li>
 (very conservative)

### results: proton scattering data



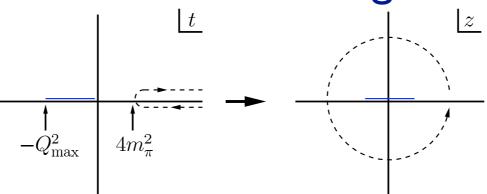
- larger  $Q^2$  range: sensitive to more coefficients in expansion, but doesn't improve slope at  $Q^2=0$ 





## <u>results</u>: proton + neutron scattering data

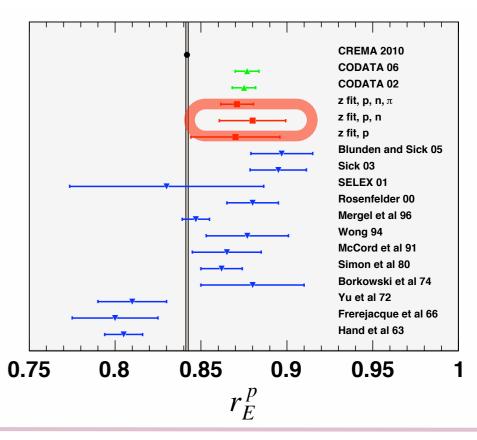
- isoscalar threshold is actually higher (  $9m_{\pi}^2$  vs  $4~m_{\pi}^2$ )
- higher threshold  $\Rightarrow$  smaller  $z \Rightarrow$  stronger constraints



- use combined fit of proton and neutron data to

decompose isoscalar and isovector

$$r_E^p = 0.880^{+0.017}_{-0.020} \pm 0.007 \, \mathrm{fm}$$
 expt shape:  $|\mathbf{a}| < 5 \rightarrow |\mathbf{a}| < 10$ 



<u>results</u>: proton + neutron scattering data, and ππ→NÑ data

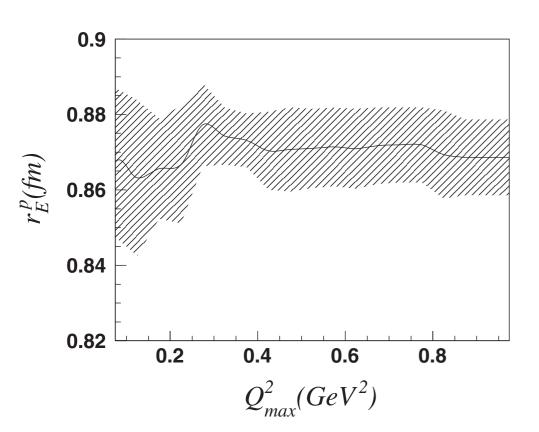
- in isovector channel, only  $\pi\pi$  states contribute below  $16m_\pi{}^2$ 

Im 
$$G_E^{(1)}(t) = \frac{2}{m_N \sqrt{t}} (t/4 - m_\pi^2)^{3/2} F_\pi(t)^* f_+^1(t)$$
, partial amplitude pion form factor for  $\Pi \Pi \to N \tilde{N}$ 

- effectively raise the isovector threshold (⇒smaller z) by including this contribution explicitly

$$G_E^{(1)}(t) = G_{\text{cut}}(t) + \sum_k a_k z^k (t, t_{\text{cut}} = 16m_{\pi}^2, t_0)$$

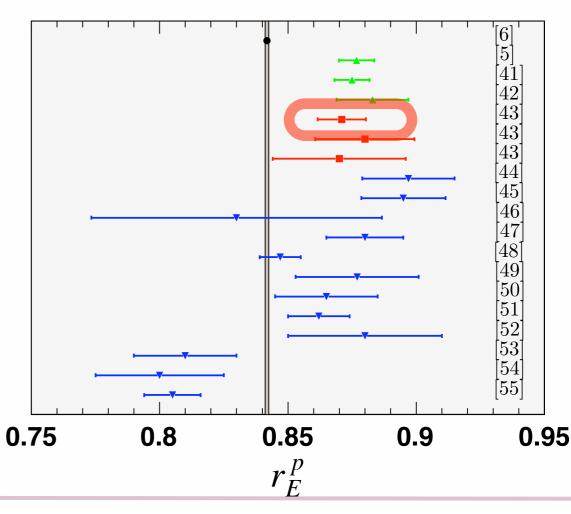
## results: proton + neutron scattering data, and ππ→NÑ data normalization error for



 $r_E^p = 0.871 \pm 0.009 \pm 0.002 \pm 0.002 \, \mathrm{fm}$  expt shape:  $|\mathbf{a}| < \mathbf{5} \rightarrow |\mathbf{a}| < \mathbf{10}$ 

- consistent results for  $r_E$  using proton, proton+neutron, proton+neutron+ $\pi\pi$ , different  $Q^2$  ranges

- leaves significant discrepancy with muonic H extraction

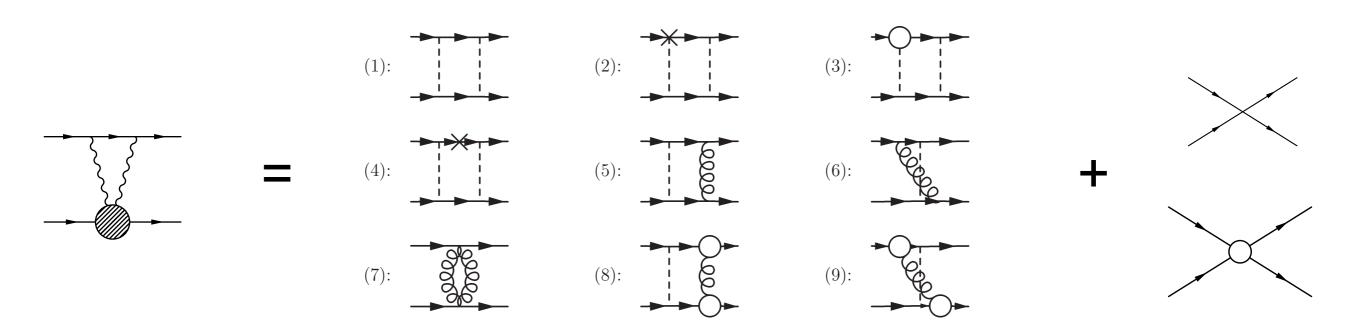


#### Investigate scheme dependence: no significant effect

TABLE III. The rms charge radius extracted using electron-proton and electron-neutron scattering data, and different schemes presented in the text. The neutron form-factor slope is constrained using (31). A cut  $Q_{\text{max}}^2 = 0.5 \text{ GeV}^2$  is enforced. In the lower part of the table, the bounds on  $\sum_k a_k^2$  from Table II are multiplied by 4.  $\phi_{\text{VMD}}$  and  $\phi_{\text{OPE}}$  are defined in Eqs. (22) and (23).

	$k_{\text{max}} = 2$	3	4	5	6
$\phi = 1,  t_0 = 0,   a_k  \le 10$	888+5	865+11	888+17	882+21	878+20
	$\chi^2 = 33.67$	23.65	21.80	21.13	20.47
$\phi = 1, t_0 = 0,  a_k  \le 5$	$888^{+5}_{-5}$	$865^{+11}_{-11}$	$881^{+10}_{-16}$	$885^{+16}_{-21}$	$882^{+18}_{-20}$
	$\chi^2 = 33.67$	23.65	21.95	21.46	21.06
$\phi = \phi_{\text{VMD}}, t_0 = 0,  a_k  \le 10$	$865^{+6}_{-6}$	$874^{+12}_{-13}$	$884^{+23}_{-24}$	$879^{+24}_{+22}$	$877^{+22}_{-20}$
	$\chi^2 = 23.26$	22.50	22.15	21.59	21.09
$\phi = 1, t_0 = 0$	888+5	865+11	880+13	882+14	882+15
	$\chi^2 = 33.67$	23.65	22.07	21.45	21.18
$\phi = \phi_{\text{OPE}}, t_0 = 0$	$904^{+5}_{-5}$	$861^{+10}_{-11}$	$888^{+14}_{-21}$	$883^{+20}_{-20}$	$881^{+20}_{-19}$
	$\chi^2 = 61.34$	24.38	21.62	20.86	20.51
$\phi = \phi_{\text{OPE}}, t_0 = t_0^{\text{opt}} (0.5 \text{ GeV}^2)$	$912^{+5}_{-5}$	$869^{+9}_{-9}$	$887^{+18}_{-19}$	$881^{+20}_{-19}$	$880^{+20}_{-19}$
	$\chi^2 = 93.69$	22.54	21.05	20.32	20.32

## NRQED part (2): two-photon exchange



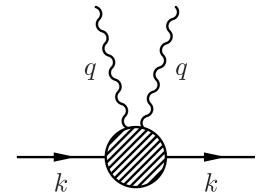
- to reproduce large momentum regions, include fourfermion counterterms

$$\mathcal{L}_{\text{ct}} = \frac{d_1}{M^2} \psi_p^{\dagger} \psi_p \psi_e^{\dagger} \psi_e + \frac{d_2}{M^2} \psi_p^{\dagger} \vec{\sigma} \psi_p \cdot \psi_e^{\dagger} \vec{\sigma} \psi_e + \dots$$

- perform matching at a convenient point, e.g. all external particles onshell, at rest: hadronic part is forward Compton amplitude

## forward Compton amplitude

- QCD input summarized by amplitudes of forward scattering



$$W^{\mu\nu}(q,k) = i \int d^4x \, e^{iq \cdot x} \langle \operatorname{proton}(k,s) | T\{J^{\mu}(x)J^{\nu}(0)\} | \operatorname{proton}(k,s) \rangle$$

$$= \bar{u}_s(k) \left\{ \left( -g^{\mu\nu} + \frac{q^{\mu}q^{\nu}}{q^2} \right) W_1(\nu, Q^2) + \left( k^{\mu} - \frac{k \cdot q}{q^2} q^{\mu} \right) \left( k^{\nu} - \frac{k \cdot q}{q^2} q^{\nu} \right) W_2(\nu, Q^2) + H_1(\nu, Q^2) \left\{ [\gamma_{\nu}, \not q] k_{\mu} - [\gamma_{\mu}, \not q] k_{\nu} + [\gamma_{\mu}, \gamma_{\nu}] k \cdot q \right\}$$

$$+ H_2(\nu, Q^2) \left\{ [\gamma_{\nu}, \not q] q_{\mu} - [\gamma_{\mu}, \not q] q_{\nu} + [\gamma_{\mu}, \gamma_{\nu}] q^2 \right\} \right\} u_s(k)$$

- four invariant functions of photon energy  $\vee$  and invariant  $\mathbb{Q}^2$ : two spin-independent, two spin-dependent
- in DIS, just interested in imaginary part, but here need the whole thing

## dispersion relations for forward amplitudes

$$W_2(\nu,Q^2) = \frac{1}{\pi} \int_0^\infty d\nu'^2 \frac{\mathrm{Im} W_2(\nu',Q^2)}{\nu'^2 - \nu^2} = W_2^{\mathrm{proton}}(\nu,Q^2) + \frac{1}{\pi} \int_{\nu_{\mathrm{cut}}}^\infty d\nu'^2 \frac{\mathrm{Im} W_2(\nu',Q^2)}{\nu'^2 - \nu^2}$$
 onshell proton form factors inelastic cross section

⇒ determined by measurable quantities

## what if the dispersion integral doesn't converge?

$$W_{1}(\nu,Q^{2}) = W_{1}(0,Q^{2}) + \frac{\nu^{2}}{\pi} \int_{\nu_{\text{cut}}^{2}}^{\infty} d\nu'^{2} \frac{\text{Im}W_{1}(\nu',Q^{2})}{\nu'^{2}(\nu'^{2}-\nu^{2})}$$
?? DIS structure function

### dispersion relation with subtraction:

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-  $W_1(v,Q^2)$  not determined by imaginary part

$$W_1(\nu, Q^2) = W_1(0, Q^2) + \frac{\nu^2}{\pi} \int_{\nu_{\text{cut}}^2}^{\infty} d\nu'^2 \frac{\text{Im}W_1(\nu', Q^2)}{\nu'^2(\nu'^2 - \nu^2)}$$

- no intrinsic meaning to "proton contribution" and "non-proton contribution" to  $W_1(0,Q^2)$ 

- can analyze at low momentum using NRQED: double scattering of proton off external static electromagnetic field

## having determined the Wilson coefficients from data, find the leading behavior of $W_1(0,Q^2)$ at small $Q^2$ :

$$W_{1}(0,Q^{2}) = 2(-1 + c_{F}^{2}) + \frac{Q^{2}}{2M^{2}} \left(c_{F}^{2} - 2c_{F}c_{W1} + 2c_{M} + c_{A1}\right) + \dots$$

$$= 2a_{p}(2 + a_{p}) + \frac{Q^{2}}{2M^{2}} \left(-8a_{p}F'_{1}(0) - 8(1 + a_{p})F'_{2}(0) + M^{3}\frac{4\pi}{e^{2}}\bar{\beta}\right) + \dots$$

$$\approx 14 + \frac{Q^{2}}{\text{GeV}^{2}} \left(-78 + 6\right) + \dots$$

## OPE expansion at large Q<sup>2</sup>:

$$W_1(0, Q^2) \approx \frac{4M^2}{Q^2} \sum_f e_f^2 \left( A_f^{(2)} - f_{T_f} \right)$$

$$\langle P(\mathbf{k})|\bar{q}_f \left(\gamma^{\{\mu}iD^{\nu\}} - \frac{1}{4}g^{\mu\nu}iD\right) q_f|P(\mathbf{k})\rangle \equiv 2A_f^{(2)} \left(k^{\mu}k^{\nu} - \frac{1}{4}g^{\mu\nu}M^2\right)$$
$$\langle P(\mathbf{k})|m_f\bar{q}_fq_f|P(\mathbf{k})\rangle \equiv 2M^2f_{T_f}$$

## How do we interpolate?

## Previous analyses use ansatz of "proton" + "non-proton"

$$W_{1}(0,Q^{2}) = 2(-1+c_{F}^{2}) + \frac{Q^{2}}{2M^{2}} \left(c_{F}^{2} - 2c_{F}c_{W1} + 2c_{M} + c_{A1}\right) + \dots$$

$$= 2a_{p}(2+a_{p}) + \frac{Q^{2}}{2M^{2}} \left(-8a_{p}F'_{1}(0) - 8(1+a_{p})F'_{2}(0) + M^{3}\frac{4\pi}{e^{2}}\bar{\beta}\right) + \dots$$

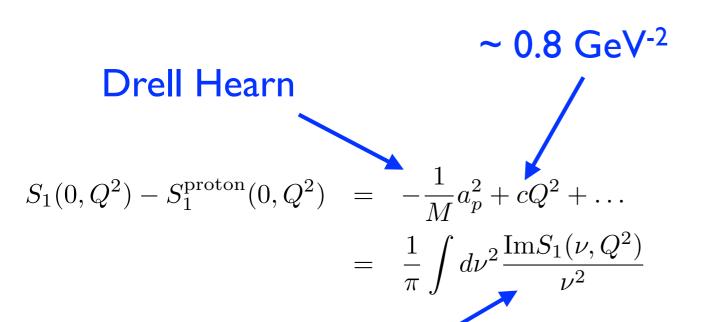
$$2F_{2}(-Q^{2})[2F_{1}(-Q^{2}) + F_{2}(-Q^{2})] \qquad \frac{4\pi}{e^{2}}\bar{\beta}\frac{MQ^{2}}{2}\frac{1}{\left(1+Q^{2}/0.71\,\text{GeV}^{2}\right)^{4}}$$

e.g. Pachucki 1999

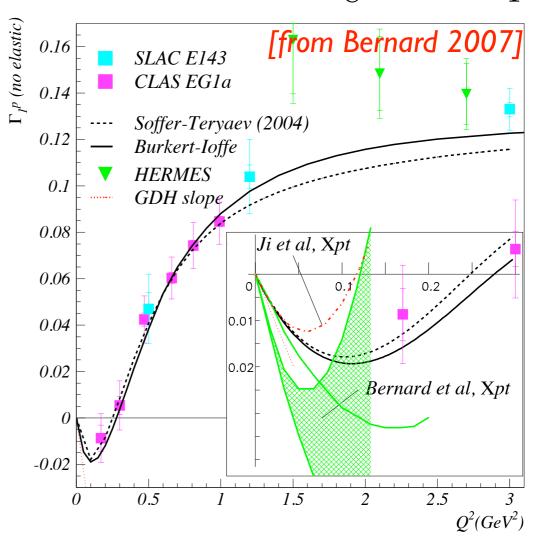
- assumes ad hoc separation into "proton" versus "non-proton" states, and assigns form factors to the former
- wrong behavior at large  $Q^2$  (  $I/Q^6$  instead of  $I/Q^2$  )

This ansatz fails dramatically for experimentally accessible spin-dependent structure function

 $\Gamma_1 = \frac{Q^2}{8} \bar{S}_1 = \frac{MQ^2}{4} \bar{H}_1$ 



data (unsubtracted dispersion relation)



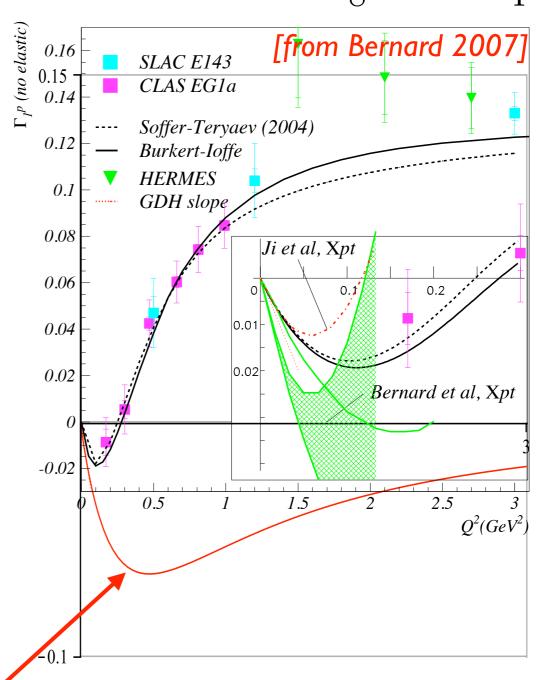
This ansatz fails dramatically for experimentally accessible spin-dependent structure function

 $\Gamma_1 = \frac{Q^2}{8} \bar{S}_1 = \frac{MQ^2}{4} \bar{H}_1$ 

Drell Hearn
$$S_{1}(0,Q^{2}) - S_{1}^{\text{proton}}(0,Q^{2}) = -\frac{1}{M}a_{p}^{2} + cQ^{2} + \dots$$

$$= \frac{1}{\pi} \int d\nu^{2} \frac{\text{Im}S_{1}(\nu,Q^{2})}{\nu^{2}}$$

data (unsubtracted dispersion relation)



SIFF (sticking in form factors) ansatz

- model independently, in small lepton mass limit, 2-photon contribution to energy is

$$\Delta E(nS) = (Z\alpha)^{5} \frac{m_{r}^{3}}{\pi n^{3}} \frac{m_{e}}{m_{p}} \left\{ \log m_{e} \left[ m_{p}^{3} \frac{4\pi}{e^{2}} \left( 5\bar{\alpha} - \bar{\beta} \right) - 3a_{p}^{2} \right] + c_{0} + c_{1} m_{e} + \dots \right\}$$

universal hadronic parameters

- unfortunately,  $m_{\mu}/m_{\pi}$ ,  $m_{\mu}/(m_{\Delta}-m_{p})$  not small

Note that low energy  $\mu p$  scattering, including radiative corrections, is predicted by NRQED: sufficient data can determine  $d_2$ 

- proposal for  $\mu^+p$ ,  $\mu^-p$  scattering (PSI, Gilman, Piasetzky et al)
- systematic computations in progress

## In absence of a measurement for this new hadronic quantity, uncertainty of ~0.004 meV is not justified

	[	Pachucki]	[Borie]	[Hill, Paz]	
Contribution		Ref. [20]	Ref. [23]	This work	
$\delta E^{ m vertex}$		-0.0099	-0.0096	-0.0108	
	$\delta E_{\mu H}^{\rm proton}$			-0.016	
$\delta E^{\mathrm{two}-\gamma}$	$\delta E_{\mu H}^{W_1(0,Q^2)}$	0.035	0.051	Model Dependent	[Carlson
	$\delta E_{\mu H}^{ m continuum}$			0.013 [19]	[Carlson, Vanderhaegen]
Total		0.025	0.042		, a.i.ee.iiiaegeiij

TABLE I: Comparison between this and previous works for  $\mathcal{O}(\alpha^5)$  proton structure corrections to the 2P-2S Lamb shift in muonic hydrogen, in meV.

## Investigate the impact of modified structure corrections and larger uncertainty on the two-photon exchange contribution

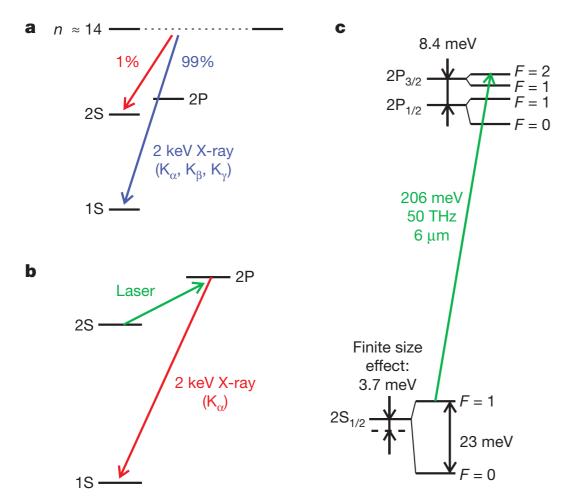
				Pohl et al compilation (Nature 2010)	RJH, Paz	reason for change
		vertex	correction	-0.0096 meV	-0.0108	mismatch in r definitions
		two ph	oton (d <sub>2</sub> )	0.051	? ~ 0.05 +/- 0.05	model dependent
		"recoil t	finite size"	0.013	0	double counting
		t	otal	210.0011(45) -5.2262r <sup>2</sup>	209.987(50) -5.2262 r <sup>2</sup>	
		extract	ed radius	0.8421(6) fm	0.841(6) fm	
Н	CC	DATA06	0.876(8)	4.2σ	3.5σ	
e-p	Si	ck 2005	0.895(18)	2.9σ	2.8σ	
	JL	ab 2011	0.875(10)	3.3σ	2.9σ	
		Mainz	0.879(8)	4.6σ	3.8σ	
H and e-p	CC	DATA I 0	0.8775(51)	6.9σ	4.6σ	
		ер	0.870(26)	Ι.Ισ	Ι.Ισ	
		ep, en	0.880(20)	Ι.9σ	Ι.9σ	
	ep,	en, ppNN	0.871(10)	2.9σ	2.6σ	

## Summary

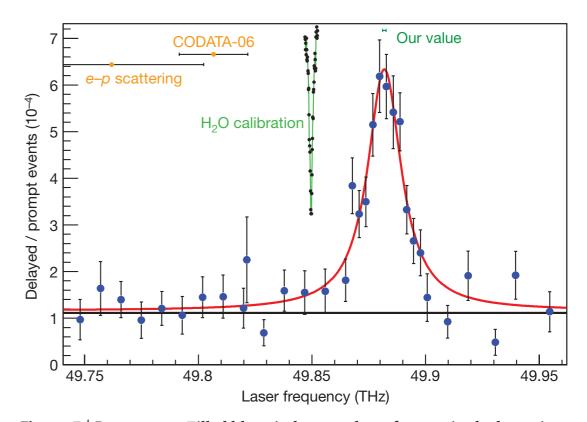
## The proton radius is still a puzzle.

- most mundane resolution may be  $\sim 5\sigma$  shift in Rydberg (less mundane resolutions postulated)
- z expansion provides model independent extrapolation for radius determination from scattering data
  - analysis should/will be implemented with most recent scattering data
  - NRQED can be used to eliminate model dependence in radiative corrections at low energy
- NRQED analysis of proton structure for hydrogenic bound states
  - model independent translation between scattering and bound state observables
  - model-dependent assumptions in previous analyses

- possibility of significant new effects in contact interaction describing two-photon exchange
  - can measure strength of this contact interaction directly in muon-proton scattering



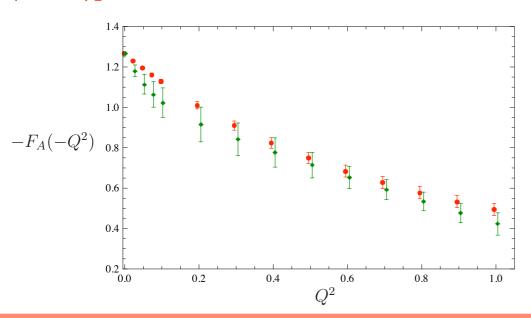
**Figure 1** | **Energy levels, cascade and experimental principle in muonic hydrogen. a**, About 99% of the muons proceed directly to the 1S ground state during the muonic cascade, emitting 'prompt' K-series X-rays (blue). 1% remain in the metastable 2S state (red). **b**, The  $\mu p(2S)$  atoms are illuminated by a laser pulse (green) at 'delayed' times. If the laser is on resonance, delayed  $K_{\alpha}$  X-rays are observed (red). **c**, Vacuum polarization dominates the Lamb shift in  $\mu p$ . The proton's finite size effect on the 2S state is large. The green arrow indicates the observed laser transition at  $\lambda = 6 \, \mu m$ .

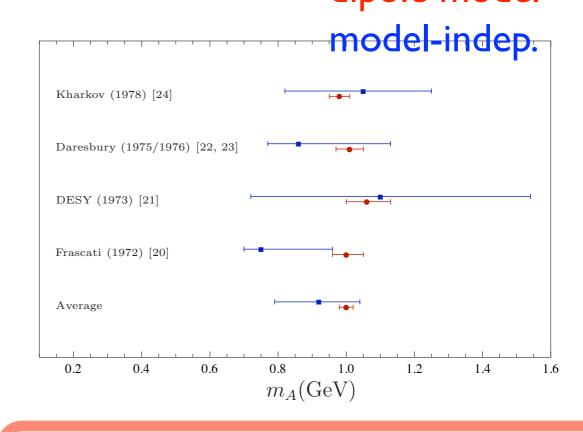


**Figure 5** | **Resonance.** Filled blue circles, number of events in the laser time window normalized to the number of 'prompt' events as a function of the laser frequency. The fit (red) is a Lorentzian on top of a flat background, and gives a  $\chi^2$ /d.f. of 28.1/28. The predictions for the line position using the proton radius from CODATA<sup>3</sup> or electron scattering<sup>1,2</sup> are indicated (yellow data points, top left). Our result is also shown ('our value'). All error bars are the  $\pm 1$  s.d. regions. One of the calibration measurements using water absorption is also shown (black filled circles, green line).

#### Aside: similar analysis for axial radius of relevance to neutrino scattering dipole model

[data from MiniBooNE, PRD81, 092005 (2010)<sub>1</sub>





$$m_A = 0.85^{+0.22}_{-0.07} \pm 0.09 \,\text{GeV}$$

(neutrino scattering)

$$m_A = 0.92^{+0.12}_{-0.13} \pm 0.08 \,\text{GeV}$$
 (electroproduction)

$$m_A^{\rm dipole} = 1.29 \pm 0.05 \,{\rm GeV}$$

$$m_A^{\text{dipole}} = 1.00 \pm 0.02 \,\text{GeV}$$

$$r_A = \begin{cases} 0.80^{+0.07}_{-0.17} \pm 0.12 \text{ fm} & \text{(neutrino scattering)} \\ 0.74^{+0.12}_{-0.09} \pm 0.05 \text{ fm} & \text{(electroproduction)} \end{cases}$$

Using default nuclear model (relativistic fermi gas), discrepancy can be attributed to incorrect form factor assumption